EFFICIENT TUITION FEES AND EXAMINATIONS

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Abstract
We assume that students observe only a private, noisy signal of their ability and that universities can condition admission decisions on the results of noisy tests. If the university observes a private signal of each student's ability, which is soft information, then asymmetries of information are two-sided, and the optimal admission policy involves a mix of pricing and pre-entry selection, based on the university's private information. In contrast, if all test results are public knowledge, then there is no sorting on the basis of test scores: Tuition alone does the job of implementing an optimal degree of student self-selection. These results do not depend on the existence of peer effects. The optimal tuition follows a classic marginal social-cost pricing rule. (JEL: D82, H42, I22, J24)

1. Introduction

Why would a university at the same time set admission standards and charge tuition? Students both pay a charge and experience demand rationing at the current price. This is an instance of the "awkward economics of higher education," analyzed by Winston (1999). Indeed, at first glance, a non-profit, welfare-maximizing university would be willing to reduce tuition, whereas a rent or profit-maximizing university would try to reduce unsatisfied demand by increasing the price. Yet, in the United States, we observe both selection practices based on admission standards and high levels of tuition. Reform plans, inspired by this model, are

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sweeping Europe (see, e.g., Jacobs and van der Ploeg 2006). A complete information setting is unable to account for this pricing puzzle, and more precisely, for the role of admission standards. Rothschild and White (1995) convincingly argue that students are used as inputs in the process of production of their own human capital. If universities were completely informed about the applicants' characteristics, the use of each student, viewed as an input in the education process, could be appropriately priced by means of tuition fees. Under the more realistic assumption of incomplete information, an explanation for this puzzle relies on the presence of peer-group effects. The presence of bright students may be especially desirable for a university. They generate positive externalities for other students and teachers and may be detected by means of tests or entrance examinations. In the presence of competition among universities to attract the best students, the existence of demand rationing may be explained as well. Universities exercise control over the quality of enrolled students by generating an excess demand, combined with selection of the most desirable applicants.[1] However, there might be other convincing and complementary explanations for this widespread combination of direct pre-entry selection and pricing in higher education.

The present contribution provides an explanation for university admission policies and pricing that doesn't rely on peer effects and competition but hinges entirely upon the presence of double-sided asymmetric information in the university enrollment process. Students are assumed to have better information than universities about some relevant aspects of their talents, such as motivation and ability to exert effort, whereas universities are assumed more able to assess certain skills along other dimensions. More precisely, admission offices use formal, quantitative, namely, "hard" information such as test scores and grade point averages, and at the same time, informal, subjective, namely, "soft" information, based on interviews, CVs, and letters of motivation, in the assessment of the expected performance of the student. Soft information is, by definition, hard to put down on paper. It is then difficult to transmit to the student, whereas there is no material difficulty to disclose hard information. In the following, we will assume that test scores are revealed to students, while subjective assessments remain private information of the university. As long as universities use soft information to sort students, in addition or not to hard information, they can form better predictions of the labor market value of some traits than the students themselves. Under these assumptions, it is optimal to combine direct screening of applications on the basis of some admission standards and self-selection by means of tuition and fees. This kind of policy mix is shown to characterize the behavior of the

1. Again, see Winston (1999). Many contributions have discussed the identification and produced estimations of these peer effects. See, for instance, among other contributions: Manski (1993b), Hoxby (2000), Sacerdote (2001), and Zimmerman (2003). The theory of peer group effects, or local interactions in education, has been studied, among other contributions, by Arnott and Rowse (1987), de Bartolome (1990), Benabou (1993), and Epple and Romano (1998).

welfare-maximizing, as well as that of the rent- or profit-maximizing universities. The bilateral asymmetric information assumption is essential, because if the faculty's information set is strictly included into that of the student, selection by means of pricing alone is optimal, and admission standards are useless. These results are obtained in the absence of borrowing constraints, and in the absence of inequality aversion on the part of universities: They are not driven by the presence of student-loan market imperfections, and they do not depend on the existence of a redistribution or insurance motive of academic authorities. The results are robust to the re-introduction of peer effects, as shown by a previous version of this paper.[2]

We formalize these ideas in a simple model of a university, which chooses a tuition level and an admission standard so as to maximize its objective function under the constraint that enrollment is equal to the number of admissible student applications. Students base their application decisions on a forecast of the labor-market skill-premium that can be acquired by means of higher education, compared to direct and opportunity costs of higher education (respectively, tuition and foregone unskilled wages). A student's future wage depends on some personal "ability" characteristic that the student doesn't know: He or she can only form an expectation of this ability term, conditional on private and public informative signals. This view is in tune with the contemporary econometric work on returns to education and the impact of tuition.[3] The faculty base their admission decision on another signal of the student's ability that they privately observe (soft information). Under these assumptions, we prove that it is optimal for the university to directly select applications on the basis of the private signal, and to set a positive tuition level. This is true under two polar assumptions about the university's objective: social surplus maximization, or profit maximization.[4] Therefore, the assumption that test scores are not perfect predictors of a student's future labor-market potential and that universities can rely on non-redundant, informative but private signals of the same potential, implies the optimality of the observed dual admission mechanism.

If there is no private signal of the university, and if students can form expectations with the help of their own private signals (and possibly of some publicly observable test results), then the optimal policy is to set the lowest possible admission standards, and to let tuition do the entire job of implementing the right amount of self-selection.

2. Gary-Bobo and Trannoy (2005).

3. Among many contributions: Keane and Wolpin (2001), Carneiro, Hanson, and Heckman (2003), Cunha, Hechman, and Navarro (2005).

4. The debate on the appropriate university objective is not settled. Some recent papers have proposed a formal analysis of this question and of the more difficult problem of imperfect university competition; see, for example, Borooah (1994), Del Rey (2001), De Fraja and Iossa (2002), and Vanhaecht and Pauwels (2005).

Our work is related to the theoretical literature on admission standards. Pioneering work on this subject is due, notably, to Costrell (1994), Betts (1998), Fernández (1998), and Fernández and Galí (1999). But these contributions have not emphasized the role of incomplete and asymmetric information as we do here. The recent work of Epple et al. (2006) models college admissions as a bargaining game between the college and the potential student under asymmetric information, and analyzes the evidence of student "profiling" practices in the college admission processes. Epple et al. confirm the importance of the private signals used by universities (i.e. different colleges may "read" student records differently), but their goal is not to provide a normative analysis of the observed admission practices. Our contribution is also related to optimal taxation theory, and more particularly to studies of optimal education policies under asymmetric information (e.g., De Fraja 2002; Fleurbaey, Gary-Boho, and Maguain 2002; Bovenberg and Jacobs 2005).

Our result that, even in absence of competition, universities should simultaneously rely on tuition fees and admission standards to select students is directly related to the present debate on tuition vs. taxation. The importance of this topic has increased with the worsening of the financial crisis of universities in many countries, the increase of tuition and fees in the US, and the discussions surrounding higher-education reform and student loans in Europe.[5]

In the following, Section 2 presents the model and basic assumptions; Section 3 presents the main results, obtained under double-sided asymmetric information; Section 4 discusses the special case in which the students are better informed than universities about their own ability; and Section 5 contains our concluding remarks. Detailed proofs are in the Appendix.

2. The Model and Basic Assumptions

To simplify the analysis, we assume that there exists a single university (or college), with a single department, and endowed with market power as a provider of higher education (and skilled workers).

2.1. Skill Premium, Ability, and Preferences

We suppose that there exist two categories of workers only, the skilled, who are graduates from the university (or college), and the unskilled, who did not study.

5. See Johnson (1984), Creedy and Francois (1990), Barr (1993), Chapman (1997), Ehrenberg (2000), and Jacobs and van der Ploeg (2006).

The unskilled workers' wage rate is a constant w_0, assumed non-random for simplicity,[6] and the university (or college) graduates' wage is a random variable denoted $\tilde{w}$. The skilled wage depends on a factor, common to all students, and on an individual factor. The common factor is the skill premium, earned by means of higher education, and denoted K. We stick to the classic human-capital theory of education, assuming that education improves productivity, and that productivity will be observed by employers.[7] The skill premium is assumed to be a continuous differentiable function of the number of graduates, denoted q. In a simple static general equilibrium model, the skill premium would be a decreasing function of the amount of skilled labour. But if the number of educated people generates an economy-wide externality of the kind described by endogenous growth theorists (cf. Aghion and Howitt 1997), then our skill premium function could become an increasing function of q. We will discuss the implications of both cases in the following. An important, and probably strong—although very common—assumption is that all agents observe and (or) form correct expectations about the skill premium K.

Each student is then characterized by the realization of a random variable denoted $\tilde{\theta}$, describing the individual's talent in skilled activities, and called "ability." This individual factor $\tilde{\theta}$ has a zero mean, and captures the fact that graduates are not all equal, due to differing talents, that are reflected in more or less brilliant grades, as well as more or less lucrative perspectives on the labor market. We will explore different assumptions relative to the information of agents about ability below; this variable is not observed, neither by the university, nor by the student, and each student receives a noisy private signal relative to her (his) own ability. The university and the students are supposed to know the probability distribution of ability.

To sum up, the following relation (i.e., a Mincer equation) describes the future wage of a skilled worker.

$$\ln(\tilde{w}) = \ln(w_0) + K(q) + \tilde{\theta}. \tag{1}$$

The student's preferences are represented by the same inter-temporal, infinite horizon, additively separable utility function. Higher education takes place in the first period of the student's life-cycle, at time 0. Utility is assumed to be quasi-linear with respect to consumption at 0. Formally, for a consumption profile

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6. We could deal with the additional complexity that a higher number of graduates may increase the productivity of the unskilled workers. This would add a positive externality of education, but would not change the main results.

7. Our theory would be compatible with imperfect observations of productivity by employers, and thus with a more general signaling theory of education. On the debate between human-capital and signalling views, see Weiss (1995).

$c = (c_0, c_1, c_2, \ldots)$ we define utility $u(c)$ by

$$u(c) = c_0 + \sum_{\tau=1}^{\infty} \left(\frac{1}{1+r}\right)^{\tau} \ln(c_\tau), \tag{2}$$

where r is a psychological interest rate, used by agents to discount future utility.

To simplify the analysis, we assume that a worker's wage is constant during her (his) entire working life. Agents do not save, and consume their wages, which are expressed in real terms. With the help of these assumptions, an agent who doesn't study remains an unskilled worker for life; his or her utility can be written, using equations (1) and (2),

$$u_0 = w_0 + \frac{\ln(w_0)}{r}. \tag{3}$$

Let p denote the tuition charges, paid at time zero. The utility u_1 of a skilled worker is random, and using (1) and (2), we obtain

$$u_1 = -p + \frac{\ln(w_0)}{r} + \frac{K(q)}{r} + \frac{\tilde{\theta}}{r}. \tag{4}$$

In the following, we denote by

$$\Delta(q) = \frac{K(q)}{r} \qquad \text{and} \qquad \theta = \frac{\tilde{\theta}}{r} \tag{5}$$

the capitalized values of the skill premium and of the ability term, respectively. The cost of human-capital investment is the sum of direct costs p and opportunity costs w_0.

2.2. *Philanthropic vs Rent-Maximizing Universities*

To describe higher education costs, we assume that there exists a continuously differentiable university cost function, denoted C, depending on the number of enrolled students q. The results obtained with this simple formulation can be extended to the case in which C also depends on education quality and on the average ability of enrolled students (i.e., on *peer effects*).[8]

We define a higher education institution as "philanthropic" when its objective is to maximize the social value (social surplus) of its education activity. It is of

8. In the working paper version of the present article (Gary-Bobo and Trannoy 2005), we studied a more complicated model in which C depends on q, on chosen education quality e, and on average student ability, denoted v, that is, $C = C(q, e, v)$. This cost function captures the impact of the often discussed "peer group effects."

course a strong assumption to assume that a university is philanthropic, but this approach will provide us with a clear benchmark, equivalent to the idea of a Pareto optimum. Because utilities are quasi-linear, social surplus maximization yields efficient policies and allocations.

The philanthropic university will be financed by subsidies (or by donations), in the case of a deficit. The required amount of public resources (or donations) is simply $d = C(q) - pq$. The amount d will be subtracted from social surplus, and balances the university budget. We therefore suppose that the share of total cost that the students of a given cohort do not pay for in the form of tuition fees will be paid in the form of (lump-sum) taxes, or contributions, made by the same (or other) agents, such as alumni donations, and so forth. There are of course more subtle relations between public pricing, public subsidies voluntary contributions, and the tax system involving political economy and redistribution problems that will not be studied in the present analysis. In particular, we assume that the social justice problems are solved by means of other redistributive tools, in the hands of independent public authorities. More precisely, we assume that equality of opportunity problems are solved in the sense that no student with the necessary talents is barred from studying because of a financial constraint. We are perfectly aware of the fact that this picture is a bit too rosy and that more sophisticated modeling work is needed on the case in which imperfections of credit markets and family-background differences create unequal opportunities. We address the borrowing constraints question in Section 4.3. In most of the following, our analysis aims at showing the simple structure of the efficient higher-education pricing problem in a "first-best" situation, which can again be seen as a benchmark.

We will of course contrast the philanthropic view with the less optimistic theory of the rent-maximizing university. Under this latter view, the university is assumed to maximize its rent R, which is the difference between the total sum of tuition charges and total university cost, that is, $R = pq - C(q)$. This rent can be understood as the amount of resources made available to finance faculty activities other than teaching. In academic systems in which teacher's careers essentially depend on research achievement, it is likely that research will be a major faculty objective (although there is of course no guarantee), so that R might as well stand for research. But R can of course easily be interpreted as profit, and our model is then that of a for-profit university.[9]

Under both views, the university is endowed with market power; this is an approximation for a situation in which, say, a centrally governed public network of universities has a quantitatively important share of the higher education market,

9. We are also perfectly aware that even our rent-maximizing university is an optimistic view, with a normative flavor, and that in reality, admission on the basis of expected talent may be compounded with considerations of social class, ethnicity, and, possibly, with the goal of indirectly maximizing alumni contributions.

but the model can as well capture the behavior of a private university with a dominant position.

3. Bilateral Asymmetric Information and Entrance Examinations

3.1. Assumptions about Information

We now start the study under the assumption that the students and faculty are asymmetrically informed. We assume that students do not observe their ability variable θ. They are endowed with incomplete knowledge of their own talent, formed with the help of noisy informative signals. Students are assumed to observe a private signal of capitalized ability θ. Let s denote the unbiased private signal of students. By definition,

$$s = \theta + \varepsilon, \tag{6}$$

where ε is an independent normal random variable with a zero mean and variance σ_ε^2.

In addition, a costless test technology, used by the faculty, provides another estimation of ability. For the sake of simplicity, the test result is modeled as a random variable, denoted z, and also defined as a noisy signal of ability:

$$z = \theta + \nu, \tag{7}$$

where ν is normally distributed, with a zero mean, variance σ_ν^2, and is independent from θ and ε. We also assume that the ability variable θ has a normal distribution, with a zero mean and variance σ_θ^2.

In the present section, we assume that signal z is not observed by the students. We fundamentally assume that the faculty have the ability of detecting things about a student's future ability (and future wage), that the student himself doesn't know. The proposed model has bilateral asymmetric information because each party is endowed with a piece of relevant information that the other party does not "observe." Signal z can be viewed as capturing an assessment—for instance, a judgment on the student—following the scrutiny of an application file, which summarizes the impressions made during an interview. We later explore other possible interpretations of this test, depending on the context and institutions.[10] The teachers are, say, more able to assess some of the student's talents (and their labor market value), but the student remains better informed about her own ambitions and tastes. The same kind of bilateral asymmetry of information may

10. In Section 4, z is interpreted as a grade-point average, and assumed publicly known: this changes the results radically, but for the time being, we consider it as private information of the faculty.

hold on insurance markets, where insurers can usually make more accurate risk assessments than the insured themselves.[11]

We have in mind that the faculty's pre-entry evaluation of an applicant is soft information that cannot be communicated (disclosed) easily, due to the examination technology. When a degree leads to high qualifications (for instance a professional or a master's degree), admission is more likely to be associated with a multi-dimensional assessment of the student's abilities. This is close to the assessment problem faced by an employer who wants to hire a young executive. A firm never discloses the reasons for which an applicant has been turned down. We try to capture the same idea here. Our favorite interpretation is that if z is not revealed to students, it is not because the faculty don't want to disclose the information, but because they cannot.[12] In other words, because SAT scores (or high-school GPAs) and other publicly known records are not perfect indicators of labor market potential, then a subjective element of valuation, based on other determinants of future performance and for which there is no sufficient summary, can play a useful role, provided of course that it has some accuracy or predictive power.

Assume that the university sets an admission standard z_0 and that a student is admitted for registration only if his or her "grade" z is greater than z_0. We assume that students are Bayesian expected utility maximizers. In other words, using a term coined by Manski (1993a), our students are "adolescent econometricians." It follows that some information is transmitted to the student by the admission decision: the student knows if the event $z \geq z_0$ is true (a coarse information about z), when the admission decision is disclosed.

Finally, if there was in addition a third signal, say, an SAT score or high-school GPA, assumed publicly known, the results would be essentially the same, because both the student and the faculty would condition on this signal to form their expectations of future performance. This would complicate the model without much benefit, and we will formally study the case of two signals only, but in the following, we can view all expectations as being conditional on the public signals, if any.

The value of higher education can now be expressed in expected terms, conditional on observed signals.

11. A driver knows a number of things, relevant to assess her risk of accident, that the insurer doesn't know, but might at the same time not be aware of other risk determinants that the insurer, who processes a lot of observations, can translate into accident probabilities with much higher precision.

12. Asking the symmetric question can also clarify this discussion. Students are not assumed able to disclose and transmit their private signal s to the admissions office; if s is high, this would be tantamount to a statement of self-confidence.

3.2. *Student Applications and Demand for Higher Education*

Using equations (4) and (5), the expected utility of higher education, conditional on admission, is obtained as

$$E(u_1 \mid s, z \geq z_0) = -p + \frac{\ln(w_0)}{r} + \Delta(q) + E(\theta \mid s, z \geq z_0). \tag{8}$$

Define the probability of admission of a student with private signal s as $\pi(s, z_0) = \Pr(z \geq z_0 \mid s)$. Then, using equations (3), (4), and (5), an individual applies for higher education if and only if

$$E(u_1 \mid s) = \pi(s, z_0) E(u_1 \mid s, z \geq z_0) + (1 - \pi(s, z_0)) u_0 \geq u_0,$$

that is, if and only if

$$\pi(s, z_0)[\Delta(q) - p - w_0 + E(\theta \mid s, z \geq z_0)] \geq 0.$$

Or, equivalently,

$$E(\theta \mid s, z \geq z_0) \geq p + w_0 - \Delta(q). \tag{9a}$$

For convenience, we define

$$\hat{\theta}(s, z_0) = E(\theta \mid s, z \geq z_0), \tag{9b}$$

$$\theta_0 = p + w_0 - \Delta(q). \tag{9c}$$

The random signal $\hat{\theta}$ is the student's rational expectation of her own ability and θ_0 is the total expected cost of education $p + w_0$, including the opportunity cost w_0, minus the discounted utility of the skill premium Δ. Threshold θ_0 is also the minimum expected ability, below which an application is not worthwhile.

With this specification, a student is enrolled if she is willing to apply and if she satisfies the requirements of the entrance selection process based on z, that is, if and only if

$$\hat{\theta} \geq \theta_0 \quad \text{and} \quad z \geq z_0.$$

If the potential student population is of size N, total enrollment q ("effective demand" for higher education) can be expressed as follows:

$$q = N \Pr(\hat{\theta} \geq \theta_0, z \geq z_0) = N P(\theta_0, z_0). \tag{10}$$

The average quality of enrolled students can be defined as

$$v(\theta_0, z_0) = E(\theta \mid \hat{\theta} \geq \theta_0, z \geq z_0). \tag{11}$$

We now compute the function $\hat{\theta}(s, z_0)$, and study some of its properties. First remark that, using the law of iterative expectations,

$$\hat{\theta} = E(\theta \mid s, z \geq z_0) = E[E(\theta \mid s, z) \mid s, z \geq z_0].$$

Furthermore, because of normality, $E(\theta \mid s, z) = \alpha s + \beta z$, where

$$\alpha = \frac{\sigma_\theta^2 \sigma_\nu^2}{\sigma_\theta^2 \sigma_\nu^2 + \sigma_\theta^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \sigma_\nu^2} \quad \text{and} \quad \beta = \frac{\sigma_\theta^2 \sigma_\varepsilon^2}{\sigma_\theta^2 \sigma_\nu^2 + \sigma_\theta^2 \sigma_\varepsilon^2 + \sigma_\varepsilon^2 \sigma_\nu^2}. \tag{12}$$

Thus,

$$\hat{\theta}(s, z_0) = \alpha s + \beta E(z \mid s, z \geq z_0). \tag{13}$$

Let $\phi(x) = (2\pi)^{-1/2} e^{-x^2/2}$ be the standard normal density and let $\Phi(x) = \int_{-\infty}^{x} \phi(u) du$ be the standard normal cumulative distribution function. Let

$$H(x) = \frac{\phi(x)}{1 - \Phi(x)} \tag{14}$$

be the hazard rate. It is well-known that $H(x_0) = E(x \mid x \geq x_0)$ because x is a standard normal random variable $\mathcal{N}(0, 1)$. Starting from equation (13), we can prove the following result (all proofs are in the appendix).

LEMMA 1.

$$\hat{\theta}(s, z_0) = \gamma s + \beta \sigma_0 H\left(\frac{z_0 - \gamma s}{\sigma_0}\right), \tag{15}$$

where

$$\gamma = \frac{\mathrm{Cov}(s, z)}{\mathrm{Var}(z)} = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2} \qquad \textit{and} \qquad \sigma_0^2 = (1 - \gamma)\sigma_\theta^2 + \sigma_\nu^2. \tag{16}$$

Lemma 1 says that a student's forecast of his (her) own ability (or future wage) is not simply the result of a regression of ability on the private signal s, that is γs; there is also an additional term $\beta \sigma_0 H$ that represents the informational impact of learning that the student's application file has been selected by the university. The last term in equation (15) says that the student feels more self-confident when he (or she) learns that he or she has been admitted by the university. Because $H(x)$ is a strictly increasing and positive function, learning that $z \geq z_0$ (i.e., admission) boosts the student's morale all the more because z_0 is itself high. From this and other facts we derive the following useful lemma.

LEMMA 2. *$\hat{\theta}(s, z_0)$ is a differentiable and strictly increasing function of s and z_0.*

The application condition $\hat{\theta} \geq \theta_0$ can be rewritten as $s \geq \rho(\theta_0, z_0)$, where, by definition, $\rho(\cdot, z_0) = \hat{\theta}^{-1}(\cdot; z_0)$ is the inverse of $\hat{\theta}$ with respect to s. The function $\rho(\theta_0, z_0)$ represents the minimum value of the private signal, below which enrollment in higher education is not profitable (we obviously have $\theta_0 = \hat{\theta}(\rho(\theta_0; z_0), z_0)$). We can therefore express the average student ability v as

$$v(\theta_0, z_0) = E(\theta \mid s \geq \rho(\theta_0, z_0), z \geq z_0),$$

$$P(\theta_0, z_0) = \Pr(s \geq \rho(\theta_0, z_0), z \geq z_0).$$

3.3. The Philanthropic View: Optimal Selection and Tuition Fees

Expected social surplus can be written as

$$W = qE(u_1 \mid \hat{\theta} \geq \theta_0, z \geq z_0) + (N - q)u_0 + pq - C(q). \tag{17}$$

Expression (17) can be interpreted as the total sum of the q graduates' expected utilities and of the remaining $(N - q)$ unskilled workers' utilities, less the university's deficit, which must be covered by public subsidies or donations. W can be maximized with respect to (q, θ_0, z_0) instead of (q, p, z_0), given that $\theta_0 = p + w_0 - \Delta(q)$. After some simplifications, we obtain

$$W = q[\Delta(q) - w_0 + v(\theta_0, z_0)] - C(q) + Nu_0, \tag{18}$$

which must be maximized with respect to (q, θ_0, z_0) subject to the constraint $q/N = P(\theta_0, z_0)$.

This maximization problem can be decomposed into two subproblems. A first subproblem is to maximize the average student quality $v(\theta_0, z_0)$ with respect to (θ_0, z_0), for fixed q. The second subproblem is then to maximize W with respect to q, given that the optimal (θ_0, z_0) have been expressed as functions of q.

The necessary conditions for an optimal pair (θ_0, z_0) (if it is finite!) are simply

$$\frac{v_\theta}{v_z} = \frac{P_\theta}{P_z} \qquad \text{and} \qquad q/N = P(\theta_0, z_0), \tag{19}$$

where subscripts denote partial derivatives, and we denote $v_\theta = \partial v/\partial\theta_0 = \partial v/\partial p$, $P_\theta = \partial P/\partial\theta_0 = \partial P/\partial p$, $v_z = \partial v/\partial z_0$, and so on. The interpretation of condition (19) is easy if one is reminded that $\partial v/\partial\theta_0 = \partial v/\partial p$; it says that the marginal rate of substitution between p and z_0 should equal their marginal rate of transformation, conditional on the fixed production target q.

Starting from this point, the analysis becomes a bit complicated; the details are provided in the Appendix. To compute the solution of equation (19) we first use a particular change of variables: $(\theta_0, z_0) \to (a, b)$, under which the maximization program becomes an "abstract statistical selection problem." This abstract problem is that of maximizing the expected value of ability θ, denoted V, knowing that two random signals x and y, correlated with θ, are above some cutoff points

a and b, and subject to the constraint that a certain proportion π_0 of the population is retained. To sum up, we want to maximize $V(a, b) \equiv E(\theta \mid x \geq a, y \geq b)$, subject to $\pi_0 = Q(a, b)$, where, by definition, $Q(a, b) = \Pr(x \geq a, y \geq b)$.

The abstract selection problem has been studied in the statistical literature. Statisticians have considered applications of this approach to agriculture (where it is known as the "optimal culling" problem) and even suggested applications to the selection of personnel (see the contributions of Birnbaum and Meyer 1953, Weiler 1959, and Beattie 1962). The problem can be formulated in a more general way as that of optimally choosing cutoff points for measurable characteristics so as to maximize the expectation of a linear function of desirable individual traits in a population, under the assumption that characteristics and traits are multi-variate normal and subject to the constraint that a given proportion π_0 of the population is retained. Computation of the moments of a multivariate truncated normal will ease the numerical solution of the problem; the formulae for these moments are known (again see Birnbaum and Meyer (1953), and Weiler (1959)).

The solution of the transformed version of equation (19) then involves the function $J(x) = H(x) - x$. It is well known that J is positive and monotonically decreasing from $+\infty$ to 0 as x varies from $-\infty$ to $+\infty$. Using these properties of J along with the lemmas and function ρ, we can prove the following proposition.

PROPOSITION 1. *Assume that $\sigma_\varepsilon > 0$ and $\sigma_\nu > 0$. Then the first-order conditions* (19) *for an optimum possess a unique, finite solution* (θ_0^*, z_0^*) *satisfying the equation*

$$\beta\sigma_0 J\left[\frac{z_0^* - \gamma\rho\left(\theta_0^*, z_0^*\right)}{\sigma_0}\right] = \alpha\sigma_1 J\left[\frac{\rho\left(\theta_0^*, z_0^*\right) - \delta z_0^*}{\sigma_1}\right], \tag{20a}$$

where

$$\gamma = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\varepsilon^2}, \qquad \sigma_0^2 = (1-\gamma)\sigma_\theta^2 + \sigma_\nu^2, \tag{20b}$$

$$\delta = \frac{\sigma_\theta^2}{\sigma_\theta^2 + \sigma_\nu^2}, \qquad \sigma_1^2 = (1-\delta)\sigma_\theta^2 + \sigma_\varepsilon^2, \tag{20c}$$

and $q/N = P(\theta_0^*, z_0^*)$. Parameters α and β are defined by equation (12) *above.*

To find the optimal solution, it is possible to first compute the value of ρ that solves equation (20a) for any given value of z_0, then determine θ_0 as $\theta_0 = \hat{\theta}(\rho, z_0)$, and finally use $q/N = P(\theta_0, z_0)$ to pin down the solution. If $\sigma_\nu = \sigma_\varepsilon$, we obtain $\alpha = \beta$, $\gamma = \delta$, and so on. Then under $\sigma_\nu = \sigma_\varepsilon$, equation (20a) implies $z_0^* = \rho(\theta_0^*, z_0^*)$, or equivalently, $\hat{\theta}(z_0^*, z_0^*) = \theta_0^*$.

The unique solution (θ_0^*, z_0^*) to the initial problem (19) determines a unique tuition fee

$$p^* = \theta_0^* + \Delta(q) - w_0.$$

Proposition 1 says that the optimum is characterized by a "policy mix": a particular combination of selection by means of money (i.e., tuition) and of direct selection of application files by means of interviews or some form of university "profiling." The use of a private assessment z of students by the faculty is justified, insofar as these assessments have some accuracy and as applicants are not fully informed about their possibilities of success and future social values as graduates. As long as the university uses a piece of soft information that is valuable, namely, is correlated to the student's future earnings, the bilateral asymmetric-information setting and its conclusion are valid. If the student's private signal conveys much more information than z, that is, if s is very precise, we find the following result.

COROLLARY 1. *If the precision of the student's private signal becomes infinite, that is, if $\sigma_\varepsilon \to 0$, then $z_0^* \to -\infty$ and tuition becomes the only selection device.*

Corollary 1 says that the weight given to interviews or student profiling methods is zero when students have a perfect knowledge of themselves. It would then be sufficient to price higher education appropriately in order to implement an optimum. In contrast, if the precision of the selection device set up by the university increases (i.e., σ_ν decreases), admission standards are raised and the role of tuition as a self-selection device becomes less important.

With the help of statistical formulas for the moments of a multivariate truncated normal, we have performed a number of numerical computations of the solution of the abstract selection problem. Figures 1 and 2 depict numerical computations of the objective function's contours $V(a, b) = v^*$ and of the constraint $Q(a, b) = \pi_0$ in the (a, b) plane.

The problem has been normalized by choosing $\sigma_\theta = 1$ and we have played with the other parameters. On both figures, the objective function's iso-utility curve is the dashed line, and the constraint is a solid line. Figure 1 depicts a symmetric solution in which $\sigma_\nu^2 = \sigma_\varepsilon^2 = 3$ and $\pi_0 = .5$, the optimal (a, b) obviously lies on the diagonal. Figure 2 depicts an asymmetric solution when $\sigma_\varepsilon^2 = 1$ and $\sigma_\nu^2 = 1.5$. The constraint is well-behaved, but the contours of V are concave curves in the (a, b) plane. The tangency point of the two curves, however, is a regular maximum: The slope of the constraint decreases more quickly than the slope of the highest feasible iso-utility curve. This regular interior solution is in sharp contrast with the results obtained when informational asymmetries are assumed one-sided, in which case an interior (i.e., finite) solution to equation (19) doesn't exist (see the following section).

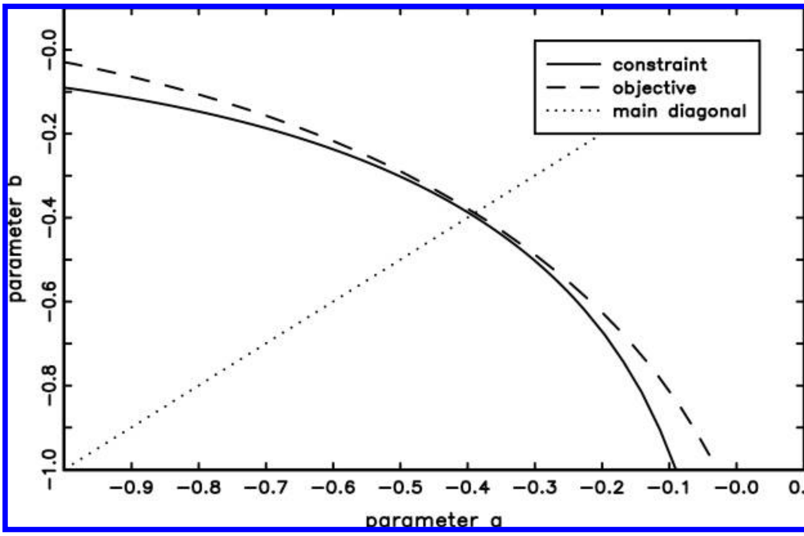

FIGURE 1. Optimal solution in the symmetric case.

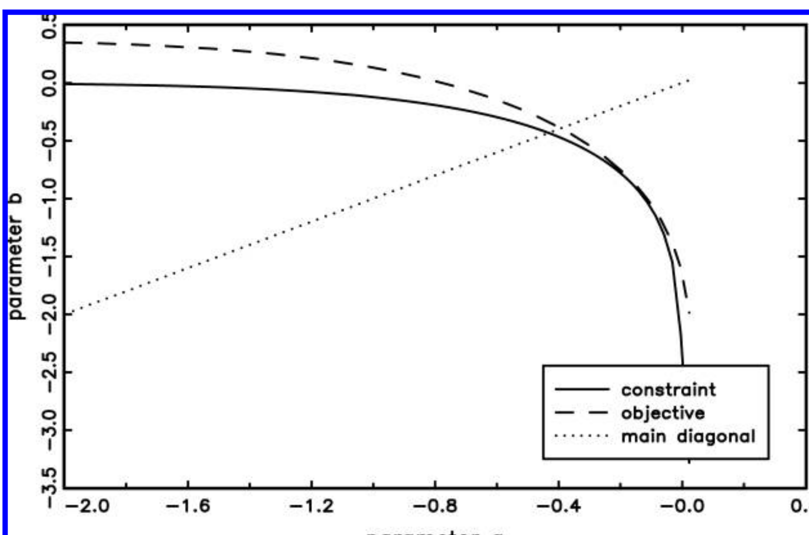

FIGURE 2. Optimal solution in the asymmetric case.

We can now solve the complete problem and choose the optimal value of enrollment q^*. The full problem involves maximizing $W = q(\Delta(q) - w_0 + v(\theta_0, z_0)) - C(q)$, with respect to (q, θ_0, z_0), subject to $q = NP(\theta_0, z_0)$. Introducing a Lagrange multiplier λ for the enrollment constraint, we find the following first-order conditions:

$$\Delta - w_0 + v + q\Delta' - C' = \lambda \qquad \text{and} \qquad \lambda = -P\frac{v_\theta}{P_\theta} = -P\frac{v_z}{P_z}, \tag{21}$$

where $v_\theta = \partial v/\partial\theta_0$, and so on. Then equation (21) implies that

$$\Delta - w_0 + v + q\Delta' - C' + Pv_\theta/P_\theta = 0.$$

Let h be defined as $v \equiv h/P$. Using first-order conditions (21) and computing the partial derivatives of v, it is easy to derive

$$\frac{v_\theta}{P_\theta} = \frac{1}{P}\left(\frac{h_\theta}{P_\theta} - v\right) = \frac{1}{P}\left(\frac{h_z}{P_z} - v\right) = \frac{v_z}{P_z}. \tag{22}$$

Substituting (22) in (21) then yields

$$\Delta - w_0 + q\Delta' - C' + (h_\theta/P_\theta) = 0.$$

Given that, by definition, $p = \theta_0 + \Delta(q) - w_0$, we find the result,

$$p^* = C'(q^*) - q^*\Delta'(q^*) + \frac{h_\theta}{P_\theta} - \theta_0. \tag{23}$$

It is not difficult to check that

$$\frac{h_\theta}{P_\theta} = E(\theta \mid s = \rho(\theta_0, z_0), z \geq z_0). \tag{24}$$

Using the law of iterative expectations, we obtain

$$\begin{aligned} h_\theta/P_\theta &= E(\theta \mid s = \rho(\theta_0, z_0), z \geq z_0) = E(\theta \mid \hat{\theta}(s, z_0) = \theta_0, z \geq z_0) \\ &= E[E(\theta \mid \hat{\theta}, s, z \geq z_0) \mid \hat{\theta} = \theta_0, z \geq z_0] \\ &= E[\hat{\theta} \mid \hat{\theta} = \theta_0, z \geq z_0] = \theta_0. \end{aligned} \tag{25}$$

We can therefore state the following proposition, which states the validity of a marginal social-cost pricing rule.

PROPOSITION 2. *The optimal enrollment q^* equates marginal social revenue $p + q\Delta'$ with marginal cost. The optimal fee satisfies*

$$p^* = C'(q^*) - q^*\Delta'(q^*). \tag{26}$$

If the skill premium does not depend on the number of graduates, that is, if $\Delta' = 0$, then we have marginal cost pricing. If an increase of the number of graduates decreases the skilled workers relative wage, that is, if $\Delta' < 0$, then the surplus-maximizing university should price above marginal cost to take the pecuniary externality $-q\Delta' > 0$ into account. If we have in mind the kind of externality described in endogenous growth theories (e.g., Aghion and Howitt 1997), then it is possible to find that $\Delta' > 0$. The university should price below marginal cost because the marginal social value of a graduate is higher than his (her) private value and can be subsidized at the optimum.

3.4. Rent-Maximizing University under Bilateral Asymmetric Information

Consider now the case in which the university maximizes the rent $R = pq - C(q)$, subject to $q = NP(\theta_0, z_0)$ and $p = \theta_0 + \Delta(q) - w_0$. This can be rewritten as the problem of maximizing $R = (\Delta(q) - w_0 + \theta_0)q - C(q)$ with respect to (q, θ_0, z_0), under the constraints $q = NP(\theta_0, z_0)$. A subproblem is to find the optimal (θ_0, z_0) for every fixed q/N in $(0, 1)$ or, equivalently, to maximize θ_0 subject to $q/N = P(\theta_0, z_0)$. This problem is not trivial. Given the definition of ρ, an equivalent form of this problem is to maximize $\theta_0 \equiv \hat{\theta}(\rho(\theta_0, z_0), z_0)$ subject to $q/N = \Pr(s \geq \rho(\theta_0, z_0), z \geq z_0)$. To have a clearer view of this problem, we use the same change of variable as above (and in the proof of proposition 1), i.e., $(\theta_0, z_0) \to (a, b)$, where $a = \rho(\theta_0, z_0)/\sigma_s$ and $b = z_0/\sigma_z$. The problem becomes that of maximizing $\hat{\theta}(\sigma_s a, \sigma_z b)$ with respect to (a, b), subject to $\pi_0 = Q(a, b)$, where by definition $Q(a, b) = \Pr(s/\sigma_s \geq a, z/\sigma_z \geq b)$ and $\pi_0 = q/N$. This is the form of our abstract statistical selection problem in the rent-maximization case: find the highest expected marginal value of ability, with respect to two cutoff points, and subject to the constraint that a certain proportion of the population is retained.

The rent-maximizing university maximizes the expected marginal value instead of the average value of ability. The first-order conditions for this problem are

$$\frac{\sigma_s \hat{\theta}_s}{\sigma_z \hat{\theta}_z} = \frac{Q_b}{Q_a} \qquad \text{and} \qquad q/N = Q(a, b). \tag{27}$$

This turns out to be a very tricky problem. First, the problem is nonconvex because the isoquants of $\hat{\theta}(\sigma_s a, \sigma_z b) =$ constant are concave in the (b, a) plane, with a very flat part when b is small (due to the asymptotic properties of H; see Lemma 1 and expression (15)). In spite of these nonstandard features, the problem typically possesses a regular global maximum as illustrated by the numerical simulation on Figure 3, which represents the global maximum in the simple symmetric case where $\sigma_\theta = \sigma_\varepsilon = \sigma_\nu = 1$ and $\pi_0 = 1/2$. The optimal value of the objective is $\theta_0^* = 0.045$.

The first-order conditions (27) typically possess other solutions that are not optimal. In particular, equation (27) can always be solved by choosing a solution,

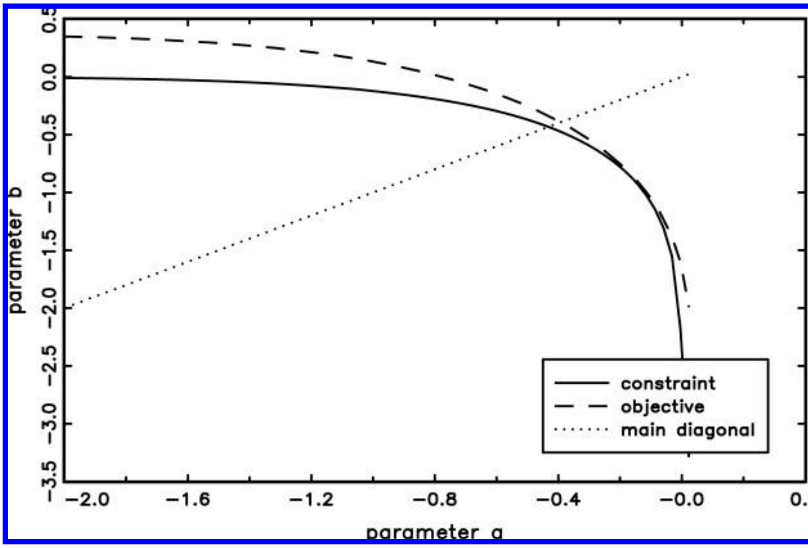

FIGURE 3. Rent maximization: optimal solution.

ad infinitum, of the form $a^{\infty} = \Phi^{-1}(1 - \pi_0)$, $b^{\infty} = -\infty$ (as shown in the proof of Proposition 3): the values of Q and $\hat{\theta}$ are finite and well defined when $b = -\infty$, but this solution is typically only a local extremum (in the symmetric example of Figure 3, the value of the objective for this latter solution is $\theta_0^{\infty} = 0$). An interior local minimum also exists (in the almost flat part of the curves, with a value $\theta_0^{\min} \approx 0.0039$ in the same symmetric example).

These results indicate that the rent-maximizing university will typically screen application files too (provided that the precision of the students' private signals is not infinite, see subsequent discussion). Why would a rent or profit maximizer fix a finite value of z_0? Because doing so increases the perceived ability of the marginal admitted student (according to Lemma 2), thus allowing an increase of tuition, because p is directly related to θ_0. This can be compared to the well-known "trendy night-club" effect (or "negative Groucho-Marx effect"): if I happen to be admitted, I feel flattered (because this means that I'm more handsome than I thought), and contrary to Groucho Marx, I'm ready to pay more to become a member of a club that accepts me.

When $\sigma_\varepsilon \to 0$, the indifference curves become very flat and almost horizontal in the (b, a) plane: it is then easy to see that (a^{∞}, b^{∞}) becomes the only optimal solution. In the Appendix, we prove the following result.

PROPOSITION 3. *If $\sigma_\varepsilon \to 0$ (the student's private signal s becomes infinitely precise), then the optimal solution of the rent maximization problem is such that $z_0 \to -\infty$.*

We can now consider the complete rent-maximization problem. Substituting the expression for $\theta_0 = \hat{\theta}(\sigma_s a, \sigma_z b)$ in the objective and using a Lagrange multiplier λ for the enrollment constraint, we obtain the following first-order conditions:

$$\Delta - w_0 + \theta_0 + q\Delta' - C' = \lambda, \tag{28a}$$

$$\lambda = -\frac{q\hat{\theta}_s\sigma_s}{NQ_a} = -\frac{q\hat{\theta}_z\sigma_z}{NQ_b}. \tag{28b}$$

Given Lemma 2 and the fact that $Q_a < 0$, we get $\lambda > 0$. Because $p = \Delta - w_0 + \theta_0$, the first-order condition is equivalent to $p = C' - q\Delta' + \lambda$. We can summarize these findings in the following proposition.

PROPOSITION 4. *The rent-maximizing university will not choose a socially optimal policy. The enrollment-tuition* (p, q) *pair will be such that* $p > C' - q\Delta'$.

The rent maximizing university maximizes the marginal student's value θ_0 instead of the average student value v (for every given value of q). This is, in essence, the reason why the rent-maximizing university makes socially inefficient decisions.

We conclude this section by emphasizing the fact that the mix of selection by means of tuition and direct screening of application files does not characterize the philantropic (i.e., welfare maximizing) university: this kind of policy mix is also rational if the university maximizes rents or profits, under bilateral asymmetric information.

4. Standard One-Sided Asymmetric Information

We now study a variant of the model in which signal z is common knowledge. The interpretation of z is now completely different; it is some official test result, like the grade-point average of a national high-school diploma (Abitur, baccalauréat, A level, etc.). The university and the student both condition their expectations with respect to z, whereas s is observed by the student only. Thus, the information set of the student includes that of the faculty. We keep the same notations as previously, as much as possible, unless otherwise specified. An individual will now apply for higher education if and only if $E(u_1|s, z) \geq u_0$. This is equivalent to $E(\theta|s, z) - p + \Delta(q) \geq w_0$ and, using $\theta_0 = p + w_0 - \Delta$, this is also equivalent to

$$\bar{\theta}(s, z) \equiv E(\theta \mid s, z) \geq \theta_0. \tag{29}$$

A student is now enrolled if and only if $z \geq z_0$ and $\bar{\theta} \geq \theta_0$. The number of enrolled students is now

$$q = N \Pr(\bar{\theta} \geq \theta_0, z \geq z_0) \equiv N P(\theta_0, z_0), \tag{30}$$

and we know that $\bar{\theta}(s, z) = \alpha s + \beta z$, as defined previously. Note that we have in fact redefined P because $\bar{\theta}$ is now replacing $\hat{\theta}$, but we keep the same notation for simplicity. The average quality of enrolled students is also redefined as follows:

$$v(\theta_0, z_0) = E(\theta \mid \bar{\theta} \geq \theta_0, z \geq z_0). \tag{31}$$

4.1. Philanthropic University under One-Sided Asymmetric Information

The welfare function is redefined as

$$W = qE(u_1 \mid \bar{\theta} \geq \theta_0, z \geq z_0) + (N - q)u_0 + pq - C(q)$$

or

$$W = q(\Delta(q) - w_0 + v(\theta_0, z_0)) - C(q) + Nu_0,$$

which is the same expression as before except that $\hat{\theta}$ is replaced with $\bar{\theta}$, and that the definitions of P and v have changed accordingly. This welfare function should be maximized with respect to (q, θ_0, z_0), subject to $q/N = P(\theta_0, z_0)$. Again, the problem can be decomposed. Let q be fixed. The necessary condition for an optimal choice of (θ_0, z_0) is $v_\theta / v_z = P_\theta / P_z$. This condition has the same expression as before, but we will show that it does not admit a finite solution.

Using the expression $v = h/P$, which defines h, and taking partial derivatives, we easily get equation (22). It is not difficult to check that this implies (see the proof of Lemma A.1) that

$$\frac{h_\theta}{P_\theta} = E(\theta \mid \bar{\theta} = \theta_0, z \geq z_0) = E(\theta \mid \bar{\theta} \geq \theta_0, z = z_0) = \frac{h_z}{P_z}. \tag{32}$$

Observe now that

$$E(\theta \mid \bar{\theta}, z) = E[E(\theta \mid \bar{\theta}, s, z) \mid \bar{\theta}, z] = E(\bar{\theta} \mid \bar{\theta}, z) = \bar{\theta}.$$

From this we derive the following property:

$$E(\theta \mid \bar{\theta} = \theta_0, z \geq z_0) = E[\bar{\theta} \mid \bar{\theta} = \theta_0, z \geq z_0] = \theta_0.$$

It follows that equation (32) becomes

$$E(\bar{\theta} \mid \bar{\theta} \geq \theta_0, z = z_0) = \theta_0. \tag{33}$$

Intuitively, this relation cannot hold unless $z_0 \to -\infty$. This can be proved rigorously and we can summarize it as Proposition 5.

PROPOSITION 5. *If z is a public signal, there does not exist a finite solution of equation* (33)*; the welfare maximizing university does not select students on the basis of test scores and $z_0 = -\infty$. Tuition is the only screening device.*

This result stands out in stark contrast with Propositions 1 and 3. If the student's information set includes that of the faculty, that is, $\{z\}$ is included into $\{s, z\}$, then it is not optimal to select the students on the basis of test scores z: tuition alone will do the job, inducing the right amount of self-selection, because students are assumed rational and are endowed with all the relevant information. In the present variant of the model, if we added the assessment of the faculty, observed (or understood) by the faculty only, and formalized as a noisy signal of the student's ability, we would find that it is useful to reject the applications of students whose signal values lie below a certain threshold. So it is really because the teachers know something that the student doesn't know about himself that it becomes optimal to select applications directly.

We can now compute the optimal price-quantity pair in this regime. Let $z_0 \to -\infty$. Define

$$\hat{P}(\theta_0) = \lim_{z_0 \to -\infty} P(\theta_0, z_0),$$

$$\hat{v}(\theta_0) = \lim_{z_0 \to -\infty} v(\theta_0, z_0) = E(\bar{\theta} \mid \bar{\theta} \geq \theta_0).$$

The optimal (p, q) maximizes $q(\Delta(q) - w_0 + \hat{v}(\theta_0)) - C(q)$ subject to $q = N\hat{P}(\theta_0)$. Using the same line of reasoning as in the proof of Proposition 2, we get the following result.

PROPOSITION 6. *If z is a public signal, the optimal tuition equates marginal social revenue with marginal cost, namely, $p = C'(q) - q\Delta'(q)$.*

For the sake of completeness, we now study the rent-maximizing university under the same informational conditions.

4.2. Rent-Maximizing University under One-Sided Asymmetric Information

The rent-seeking university is assumed to maximize $(\theta_0 + \Delta(q) - w_0)q - C(q)$ subject to $q = NP(\theta_0, z_0)$, where $P = \Pr(\bar{\theta} \geq \theta_0, z \geq z_0)$. For any fixed q/N, the university will maximize θ_0. This implies $z_0 = -\infty$ because z_0 has no value for the rent-maximizing university in this case, in sharp contrast with the bilateral asymmetric information case. To see why this case is different from the analogous problem of Section 3, note that the first-order conditions for this problem yield $P_z/P_\theta = 0$. Because $P_\theta = -\int_{z_0}^{\infty} \psi(\theta_0, z)dz$ and $P_z = -\int_{\theta_0}^{\infty} \psi(\bar{\theta}, z_0)d\bar{\theta}$, where ψ is the appropriate bivariate Gaussian density, we see that this is impossible except if $z_0 = -\infty$.

So again let $\hat{P}(\theta_0) = \lim_{z_0 \to -\infty} P(\theta_0, z_0)$. Using this definition, we obtain the following first-order condition for rent maximization:

$$p = C'(q) - q\Delta'(q) - \frac{q}{N\hat{P}_\theta}.$$

$\hat{P}_\theta < 0$ implies that, in this case, $p > C'(q) - q\Delta'(q)$.

4.3. On Borrowing Constraints under One-Sided Asymmetric Information

The results obtained under the assumption of one-sided asymmetric information are appealing, because they show that tuition is the only efficient tool when students have more ex ante information about their talents than universities, but they are not robust to the introduction of some market imperfections. Credit market or student-loan market imperfections are a natural example in this setting. We will construct a very simple model of borrowing constraints, assuming that a student's assets are independent from his (her) ability, and show that the presence of some liquidity-constrained students is enough to justify the recourse to selection on the basis of test scores, even if these scores are publicly known.

Some students, due to borrowing constraints, would not be able to pay the tuition fee, in spite of having received very good signals relative to their future ability and earnings. Because of asymmetric information, a banker would not lend enough money to a student without collateral.

For simplicity, assume that the student's "initial financial endowment" (or "asset") is a normal random variable t with a finite mean and variance. Assume that t is independent of every other random source in the model. We will now use the simplest of all models of a banker, which is sufficient to show how results change with borrowing constraints (a satisfactory model of the banker would of course be more sophisticated). Assume that if $p > t$, a student must borrow $p - t$, and that the lender will finance the student's education project if and only if $p \leq t + \Delta$. The model would of course be more realistic if the banker had her own rational assessment of θ, but our goal is to show that the slightest imperfection of the loan market is sufficient to destroy the result that the optimal $z_0 = -\infty$. This liquidity constraint can be expressed as

$$t \geq p - \Delta \equiv t_0, \tag{34}$$

or simply $t \geq t_0$. We assume that t has a positive density on the real line (t can be viewed as normal, to fix ideas). Assume in addition that, as in the asymmetric information model, t, z, and s are observed by the students and that the university observes z only. Given independence, the demand for education is now

$$q = N\tilde{P}(\theta_0, z_0, t_0) = N\,\Pr(\bar{\theta} \geq \theta_0, z \geq z_0)\,\Pr(t \geq t_0) = NP(\theta_0, z_0)\,\Pr(t \geq t_0). \tag{35}$$

With the addition of the borrowing constraint, again because of independence, the average ability of students does not change, that is,

$$v(\theta_0, z_0) = E(\theta \mid \bar{\theta} \geq \theta_0, z \geq z_0, t \geq t_0) = E(\theta \mid \bar{\theta} \geq \theta_0, z \geq z_0). \tag{36}$$

It follows that the objective can still be expressed as

$$W = q(\Delta(q) - w_0 + v) + N\bar{u}_0 - C(q),$$

where $q = N\tilde{P}(\theta_0, z_0, t_0)$. A difference with the analysis of Section 3 is that t_0 depends on θ_0. To see this, recall that $p = \theta_0 + \Delta - w_0$, so that $t_0 \equiv \theta_0 - w_0$, and one should keep in mind that $\partial t_0/\partial\theta_0 = 1$.

The optimal screening policy (θ_0, z_0) maximizes $v(\theta_0, z_0)$ for fixed q, subject to $q = N\tilde{P}(\theta_0, z_0, t_0)$ and $t_0 = \theta_0 - w_0$. We get the following first-order optimality condition, using the fact that $\tilde{P}_\theta(\theta_0, z_0, t_0) = P_\theta(\theta_0, z_0)\,\Pr(t \geq t_0)$:

$$\left(\frac{\tilde{P}_\theta}{\tilde{P}_\theta + \tilde{P}_t}\right)\frac{v_\theta}{P_\theta} = \frac{v_z}{P_z}, \tag{37}$$

where $\tilde{P}_t = \partial\tilde{P}/\partial t_0$ and $\tilde{P}_\theta = \partial\tilde{P}/\partial\theta_0$. Because t has a non-zero density, then $\tilde{P}_t < 0$ and $\tilde{P}_\theta < 0$. Clearly, $0 < \tilde{P}_\theta/(\tilde{P}_\theta + \tilde{P}_t) < 1$. Using equation (22), we find that (37) is equivalent to

$$\left(\frac{\tilde{P}_\theta}{\tilde{P}_\theta + \tilde{P}_t}\right)\frac{h_\theta}{P_\theta} + \left(\frac{\tilde{P}_t}{\tilde{P}_\theta + \tilde{P}_t}\right)v = \frac{h_z}{P_z}. \tag{38}$$

Using equation (32) and $h_\theta/P_\theta = \theta_0$, we get the still equivalent

$$\left(\frac{\tilde{P}_\theta}{\tilde{P}_\theta + \tilde{P}_t}\right)\theta_0 + \left(\frac{\tilde{P}_t}{\tilde{P}_\theta + \tilde{P}_t}\right)E(\bar{\theta} \mid \bar{\theta} \geq \theta_0, z \geq z_0) = E(\bar{\theta} \mid \bar{\theta} \geq \theta_0, z = z_0). \tag{39}$$

Equation (39) is a generalization of equation (33), but there is no obvious reason for which it should imply $z_0 = -\infty$.

Suppose on the contrary that $z_0 \to -\infty$. Then the right-hand side of equation (39) converges towards θ_0 (as shown in the proof of Proposition 5); it is easy to check that

$$\lim_{z_0 \to -\infty} v = E(\bar{\theta} \mid \bar{\theta} \geq \theta_0) \quad \text{and} \quad 0 < \lim_{z_0 \to -\infty}\left(\frac{\tilde{P}_\theta}{\tilde{P}_\theta + \tilde{P}_t}\right) < 1.$$

These limiting properties and equation (39) imply together that

$$0 = E(\bar{\theta} \mid \bar{\theta} \geq \theta_0) - \theta_0,$$

which is impossible because $\bar{\theta}$ has a normal distribution (and $J > 0$). We conclude that equation (39) cannot be solved trivially by setting $z_0 = -\infty$. Thus, we can state the following proposition.

PROPOSITION 7. *If $q > 0$ then the solution (θ_0, z_0) of the system comprising the first-order condition* (39) *and the constraint $q = N\tilde{P}(\theta_0, z_0, \theta_0 - w_0)$ is finite, if it exists; in particular, $z_0 = -\infty$ is not a solution of the system.*

The addition of borrowing constraints justifies the recourse to some form of selection on the basis of public test scores combined with tuition charges.

We have found two cases in which a policy mix of pre-entry selection and pricing can be justified: It is either because universities have knowledge about student abilities that students themselves do not have (the case of bilateral asymmetric information, where selection by means of "interviews" is combined with tuition charges), or because some good students are liquidity constrained, even if students and the university exploit the information conveyed by test scores rationally. In the latter case, it will be optimal to raise admission standards z_0 and hence simultaneously decrease tuition p (i.e., decrease θ_0) to reduce the number of poor-but-talented students inefficiently deterred by price.

5. Conclusion

We have shown that tuition and fees and direct selection of applications on the basis of tests and admission standards are both used as screening instruments in an optimal university policy. This result holds if asymmetric information is bilateral, in the sense that both the university and students possess useful, non-redundant private information about students' abilities. Our theory describes how these two screening instruments should be combined. Tuition should cover marginal cost if an increased number of graduates decreases the student's skill-premium on the labor market. If we assume that the university has no private information about potential students, that is, if the student's information set includes that of the faculty, then admission standards are useless: Tuition alone can be optimally used as a self-selection device. This latter result, however, does not remain true if there are imperfections of the student-loan market and if borrowing constraints are binding for some students. Rent-seeking or for-profit institutions will typically set prices too high with respect to the efficient tuition, which is given by a classical marginal social-cost pricing rule. However, the policy-mix result is not purely normative: It also holds if the university maximizes profit or maximizes a rent used to fund research activities. In this paper, it was assumed that the tuition fees were the same for all students. The question of discriminatory fees, that would be contingent on test scores or on the university's private signal, is not addressed here, albeit it clearly represents a realistic extension of the model. Pursuing the analysis in this direction is an avenue for further research.

Appendix: Proofs

A.1. Proof of Lemma 1

We have established that $\hat{\theta} = \alpha s + \beta E(z \,|\, s, z \geq z_0)$. Let the theoretical regression of z on s be $z = \gamma s + \zeta$ where ζ is independent of s, $\mathrm{Var}(\zeta) = \sigma_0^2$, and γ is given by equation (20). Then,

$$E(z \mid s, z \geq z_0) = \gamma s + E(\zeta \mid z \geq z_0, s).$$

Because $s \perp \zeta$, we can write

$$E(\zeta \mid z \geq z_0, s) = E(\zeta \mid z \geq z_0) = E(\zeta \mid \zeta \geq \zeta_0),$$

where $\zeta_0 = z_0 - \gamma s$. Because ζ/σ_0 is a standard normal $\mathcal{N}(0, 1)$, we obtain

$$E(z \mid s, z \geq z_0) = \gamma s + \sigma_0 E(\zeta/\sigma_0 \mid \zeta \geq \zeta_0) = \gamma s + \sigma_0 H\left(\frac{\zeta_0}{\sigma_0}\right).$$

Putting these results together, we find that

$$\hat{\theta}(s, z_0) = (\alpha + \beta\gamma)s + \beta\sigma_0 H\left(\frac{z_0 - \gamma s}{\sigma_0}\right).$$

Finally, it is easy to check that $\gamma = \alpha + \beta\gamma$.

A.2. Proof of Lemma 2

Differentiability is obvious because H is differentiable. Using $\alpha + \beta\gamma = \gamma$, we can also write

$$\hat{\theta}(s, z_0) = \alpha s + \beta z_0 + \beta\sigma_0 J\left(\frac{z_0 - \gamma s}{\sigma_0}\right),$$

where, by definition, $J(x) = H(x) - x$. It is well known that $J(x)$ is a strictly decreasing function of x. It follows that

$$\frac{\partial\hat{\theta}}{\partial s} = \alpha - \gamma\beta J'\left(\frac{z_0 - \gamma s}{\sigma_0}\right) > 0.$$

Finally, $\hat{\theta}$ is an increasing function of z_0 because H is increasing.

A.3. Proof of Proposition 1

Preliminary Step: Change of Variables. Introducing the standardized variables,

$$x = \frac{s}{\sigma_s} \qquad \text{and} \qquad y = \frac{z}{\sigma_z}, \tag{A.1}$$

where $\sigma_s^2 = \sigma_\theta^2 + \sigma_\varepsilon^2$, and $\sigma_z^2 = \sigma_\theta^2 + \sigma_\nu^2$, we can redefine v as follows:

$$v(\theta_0, z_0) = E(\theta \mid x \geq a, y \geq b) \equiv V(a, b), \tag{A.2}$$

where

$$a = \frac{\rho(\theta_0, z_0)}{\sigma_s} \qquad \text{and} \qquad b = \frac{z_0}{\sigma_z}. \tag{A.3}$$

The change of variables defined by equation (A3) is one to one because (θ_0, z_0) are obtained as a function of (a, b) as follows: $\theta_0 = \hat{\theta}(\sigma_s a, \sigma_z b)$ and $\sigma_z b = z_0$. Thus, to solve our student-ability maximization problem, we can as well maximize $V(a, b)$ with respect to (a, b), subject to the constraint $\pi_0 = Q(a, b)$, where by definition, $\pi_0 = q/N$ and

$$Q(a, b) = \Pr(x \geq a, y \geq b) = P(\theta_0, z_0). \tag{A.4}$$

This optimization problem has two variables, the abstract cutoff points (a, b), and four parameters $(\sigma_\theta^2, \sigma_\varepsilon^2, \sigma_\nu^2, \pi_0)$.

First-order conditions. The first-order conditions for an optimal solution (a^*, b^*) are simply

$$\frac{V_a}{Q_a} = \frac{V_b}{Q_b} \qquad \text{and} \qquad \pi_0 = Q(a, b). \tag{A.5a}$$

LEMMA A.1. *The first-order condition (A.5a) is equivalent to*

$$E(\theta \,|\, x = a, y \geq b) = E(\theta \,|\, x \geq a, y = b) \qquad \textit{and} \qquad \pi_0 = Q(a, b). \tag{A.5b}$$

We also have

$$\frac{V_a}{Q_a} = \frac{(g_a - V)}{Q} = \frac{(g_b - V)}{Q} = \frac{V_b}{Q_b}, \tag{A.5c}$$

where we define

$$g_a(a, b) \equiv E(\theta \mid x = a, y \geq b) \quad \textit{and} \quad g_b(a, b) \equiv E(\theta \mid x \geq a, y = b). \tag{A.5d}$$

Proof. Note first that $V(a, b) = h(a, b)/Q(a, b)$, where, by definition,

$$h(a, b) = \int_a^\infty \int_b^\infty \int_{-\infty}^\infty \theta\psi(\theta, x, y)d\theta dy dx,$$
$$Q(a, b) = \int_a^\infty \int_b^\infty \int_{-\infty}^\infty \psi(\theta, x, y)d\theta dy dx,$$

and where $\psi(\theta, x, y)$ is the multivariate normal density of (θ, x, y). With these definitions, using subscripts to denote partial derivatives with respect to a and b, we obtain, $V_a = (h_a Q - h Q_a)/Q^2$ and

$$V_a/Q_a = (1/Q)[(h_a/Q_a) - V] = V_b/Q_b = (1/Q)[(h_b/Q_b) - V].$$

This proves (A.5c) if we define $g_a = h_a/Q_a$, and $g_b = h_b/Q_b$. Thus, condition (A.5a) is equivalent to $g_a = h_a/Q_a = h_b/Q_b = g_b$. However,

$$g_a = \frac{-\int_b^\infty \int_{-\infty}^\infty \theta\psi(\theta, a, y)d\theta dy}{-\int_b^\infty \int_{-\infty}^\infty \psi(\theta, a, y)d\theta dy} = E(\theta \mid x = a, y \geq b)$$

and, similarly, $g_b = E(\theta \mid x \geq a, y = b)$, which proves (A.5b). □

To solve (A.5a), we compute g_a and g_b, and find a relationship involving the function $J(x) = H(x) - x$. Because $H(x_0) = E(x \mid x \geq x_0) > x_0$ for every finite x_0 when x is normal and because $H(x)$ tends towards x as $x \to +\infty$, $J(x)$ is always positive and decreases monotonically from $+\infty$ to 0 as x varies from $-\infty$ to $+\infty$. Using these properties of J, along with the lemmata, we can prove Proposition 1 as follows.

Final Steps. Using $E(\theta|s, z) = \alpha s + \beta z$, we obtain with (A.1) that $E(\theta|x, y) = \alpha\sigma_s x + \beta\sigma_z y$. Therefore,

$$V = E(\theta \mid x \geq a, y \geq b) = \alpha\sigma_s X(a, b) + \beta\sigma_z Y(a, b),$$

where, by definition,

$$X(a, b) = E(x \mid x \geq a, y \geq b) = \frac{k(a, b)}{Q(a, b)},$$
$$Y(a, b) = E(y \mid x \geq a, y \geq b) = \frac{l(a, b)}{Q(a, b)},$$

and obviously, also by definition,

$$k(a, b) = \int_a^\infty \int_b^\infty x\psi(x, y)dxdy \quad \text{and} \quad l(a, b) = \int_a^\infty \int_b^\infty y\psi(x, y)dxdy,$$

$\psi(x, y)$ being the bivariate normal density of (x, y). With these definitions and the expression of V, we easily get the partial derivative

$$V_a = \frac{Q_a}{Q}\left(\frac{\alpha\sigma_s k_a + \beta\sigma_z l_a}{Q_a} - V\right).$$

Furthermore,

$$\frac{k_a}{Q_a} = \frac{\int_b^\infty a\psi(a, y)dy}{\int_b^\infty \psi(a, y)dy} = a$$

and

$$\frac{l_a}{Q_a} = \frac{\int_b^\infty y\psi(a, y)dy}{\int_b^\infty \psi(a, y)dy} = E(y \mid x = a, y \geq b).$$

The expression of V_b is similar, mutatis mutandis. An equivalent form of the first-order condition (A.5a) is therefore

$$\alpha\sigma_s a + \beta\sigma_z E(y \mid x = a, y \geq b) = \alpha\sigma_s E(x \mid x \geq a, y = b) + \beta\sigma_z b,$$

or, equivalently,

$$\beta\sigma_z[E(y \mid x = a, y \geq b) - b] = \alpha\sigma_s[E(x \mid x \geq a, y = b) - a]. \tag{A.6}$$

We now use the theoretical regressions of s on z and of z on s, that is, $s = \delta z + \xi$ and $z = \gamma s + \zeta$, with regression coefficients δ and γ defined by equations (20b) and (20c). Substituting the theoretical decomposition of z, we obtain

$$\begin{aligned}
&bE(y \mid x = a, y \geq b)\\
&\quad= E\left(\frac{\gamma s + \zeta}{\sigma_z}\,\middle|\, s = \sigma_s a, \frac{\gamma s + \zeta}{\sigma_z} \geq b\right)\\
&\quad= \frac{\gamma\sigma_s}{\sigma_z}E(x \mid x = a, \zeta \geq b\sigma_z - \gamma\sigma_s a) + \frac{1}{\sigma_z}E(\zeta \mid x = a, \zeta \geq b\sigma_z - \gamma\sigma_s a)\\
&\quad= \frac{\gamma\sigma_s}{\sigma_z}a + \frac{1}{\sigma_z}E(\zeta \mid \zeta \geq \zeta_0),
\end{aligned}$$

where $\zeta_0 = b\sigma_z - \gamma\sigma_s a$, and the last line follows from the fact that $\zeta \perp s$ or $\zeta \perp x$. Similar reasoning yields the expression

$$E(x \mid x \geq a, y = b) = \frac{\delta\sigma_z}{\sigma_s}b + \frac{1}{\sigma_s}E(\xi \mid \xi \geq \xi_0),$$

where $\xi_0 = a\sigma_s - \delta\sigma_z b$.

Substituting these expressions in the first-order condition (A.6) yields

$$\beta[\gamma\sigma_s a - \sigma_z b + E(\zeta \mid \zeta \geq \zeta_0)] = \alpha[\delta\sigma_z b - \sigma_s a + E(\xi \mid \xi \geq \xi_0)]$$

or, equivalently,

$$\beta[E(\zeta \mid \zeta \geq \zeta_0) - \zeta_0] = \alpha[E(\xi \mid \xi \geq \xi_0) - \xi_0]. \tag{A.7}$$

ζ and ξ are normal random variables with a zero mean and variances $\mathrm{Var}(\zeta) = \sigma_0$ and $\mathrm{Var}(\xi) = \sigma_1$, where σ_0 and σ_1 are defined by equations (20b), and (20c). Using the definition $J(x) = H(x) - x$ and the change of variables (A.3) again, it is easy to check that expression (A.7) is equivalent to (20a).

It remains to prove that there exists a unique (a^*, b^*) that solves equation (A.7) (or (20a), up to a change of variables). To show this, remark that (A.7) can be rewritten $I(a, b) = (\alpha\sigma_1)/(\beta\sigma_0)$, where, by definition, $I(a, b) \equiv J(\zeta_0/\sigma_0)/J(\xi_0/\sigma_1)$. We first show that, for any given b, there exists a unique $a^*(b)$ which solves equation (A.7). We check that I is increasing with respect to a for every b. Compute the derivative

$$I_a = \frac{\partial I}{\partial a} = -\frac{\gamma\sigma_s J(\xi_0/\sigma_1)J'(\zeta_0/\sigma_0)}{\sigma_0 J(\xi_0/\sigma_1)^2} - \frac{\sigma_s J'(\xi_0/\sigma_1)J(\zeta_0/\sigma_0)}{\sigma_1 J(\xi_0/\sigma_1)^2}.$$

Because $J(x) > 0$ and $J' = H' - 1$, and given the well-known property $0 < H'(x) < 1$ for all x, we get $J' < 0$ and thus $I_a > 0$. In addition, $\lim_{x\to-\infty} J(x) = +\infty$ and $\lim_{x\to+\infty} J(x) = 0$.

This implies both $\lim_{a\to+\infty} I(a, b) = +\infty$ and $\lim_{a\to-\infty} I(a, b) = 0$, for every b. By the intermediate value theorem, for every b and every $(\alpha, \beta, \sigma_0, \sigma_1) > 0$ there exists a $a^*(b)$ that solves $I(a, b) = (\alpha\sigma_1)/(\beta\sigma_0)$ and thus solves equation (20a). The solution is unique because I is a strictly increasing, differentiable function of a. By the implicit function theorem, we have,

$$\frac{da^*(b)}{db} = -\frac{I_b}{I_a} > 0$$

because $I_b < 0$. Thus, a^* is an increasing function of b. We conclude that there exists a unique b such that $\pi_0 = Q(a^*(b), b)$, because $Q(a^*(b), b)$ is decreasing with respect to b and $0 < \pi_0 < 1$. We have shown the existence of a unique solution to (A5a) and, with a change of variables, to equation (20a).

A.4. Proof of Corollary 1

Note that $\sigma_\varepsilon \to 0$ implies $\alpha \to 1$, $\beta \to 0$, $\gamma \to 1$, $\sigma_0 \to \sigma_\nu > 0$, and $\sigma_1^2 \to (1-\delta)\sigma_\theta^2 > 0$. Using the notation of the proof of Proposition 1, equation (20a) becomes $1/I(a, b) = \beta\sigma_0/\alpha\sigma_1 = 0$. This implies $b = -\infty$ and $a = \Phi^{-1}(1 - q/N)$.

A.5. Proof of Proposition 3

Using Lemma 1, we can compute the derivatives

$$\hat{\theta}_s = \frac{\partial \hat{\theta}}{\partial s} = \gamma \left[1 - \beta H' \left(\frac{z_0 - \gamma s}{\sigma_0} \right) \right] \quad \text{and} \quad \hat{\theta}_z = \frac{\partial \hat{\theta}}{\partial z_0} = \beta H' \left(\frac{z_0 - \gamma s}{\sigma_0} \right). \tag{A.8}$$

Substituting these expressions in the first-order condition (27) yields,

$$\frac{Q_b(a,b)}{Q_a(a,b)} = \frac{\sigma_z \beta H' \left(\frac{\sigma_z b - \gamma \sigma_s a}{\sigma_0} \right)}{\sigma_s \gamma \left[1 - \beta H' \left(\frac{\sigma_z b - \gamma \sigma_s a}{\sigma_0} \right) \right]}. \tag{A.9}$$

Define the implicit function $b(a)$ such that $q/N \equiv Q(a, b(a))$. It is easy to check that if $a \to -\infty$, then $b(a) \to b^{\max} = \Phi^{-1}(1 - q/N)$, and if $a \to a^{\max} = \Phi^{-1}(1 - q/N)$, then $b(a) \to -\infty$. When b is replaced by $b(a)$ in (A9), the left-hand side of equation (A.9) becomes a function of a, that is, $L(a) \equiv Q_b(a, b(a))/Q_a(a, b(a))$, and the right-hand side of equation (A.9) also becomes a function of a, denoted $M(a)$. Note that $Q_b = -\int_a^{+\infty} \psi(x, b)dx$ and $Q_a = -\int_b^{+\infty} \psi(a, y)dy$, where $\psi(x, y)$ is the bivariate normal density of (x, y). It is then easy to check that $\lim_{a \to -\infty} L(a) = +\infty$ and $\lim_{a \to a^{\max}} L(a) = 0$. If $\sigma_\varepsilon > 0$, then $\beta > 0$ and $0 < \gamma < 1$. Since $\lim_{x \to +\infty} H'(x) = 1$ and $\lim_{x \to -\infty} H'(x) = 0$, we easily find that $M(a)$ decreases from $\sigma_z \beta / \sigma_s \gamma (1 - \beta) > 0$ to 0 when a goes from $-\infty$ to $a^{\max}$. This proves that equation (A.9) can always be solved, under the constraint $q/N = Q(a, b)$, by choosing $a = a^{\max}$ and $b = -\infty$. Functions Q and H, and thus $\hat{\theta}$ take well-defined values when $b = -\infty$.

Suppose now that $\sigma_\varepsilon \to 0$. Then, using equations (12) and (20b)–(20c), it is easy to check that $\sigma_s \to \sigma_\theta$, $\beta \to 0$ and $\gamma \to 1$. Because $0 \leq H' \leq 1$, the partial derivatives of $\hat{\theta}$ converge uniformly, namely, $\hat{\theta}_s \to 1$ and $\hat{\theta}_z \to 0$ when $\sigma_\varepsilon \to 0$. It follows that if $0 < (q/N) = Q(a, b) < 1$, and if equation (A.9) holds, then it must be that $Q_b \to 0$ because $0 \leq -Q_a \leq \max_a \phi(a)$. Thus, $b \to -\infty$ when $\sigma_\varepsilon \to 0$: any solution of equation (A.9) must possess this property.

A.6. Proof of Proposition 5

We must show that equation (33b) has no finite solution. Observe first that

$$\begin{aligned} E(\theta \mid \bar{\theta} \geq \theta_0, z = z_0) &= E(\bar{\theta} \mid \bar{\theta} \geq \theta_0, z = z_0) \\ &= \alpha E(s \mid \alpha s + \beta z \geq \theta_0, z = z_0) + \beta z_0. \end{aligned}$$

Let $s = \delta z + \xi$ be the theoretical regression of s on z, where $\delta = \mathrm{Cov}(s, z)/\mathrm{Var}(z)$ and ξ is independent from z with variance σ_1. Substituting this decomposition in the above expression yields

$$E(\theta \mid \bar{\theta} \geq \theta_0, z = z_0) = (\alpha\delta + \beta)z_0 + \alpha E\left(\xi \,\middle|\, \xi \geq \frac{\theta_0 - (\alpha\delta + \beta)z}{\alpha}, z = z_0\right).$$

Using the fact that $\alpha\delta + \beta = \delta$, and the fact that, by definition, $\xi \perp z$, we have

$$E(\theta \mid \bar{\theta} \geq \theta_0, z = z_0) = \delta z_0 + \alpha E\left(\xi \,\middle|\, \xi \geq \frac{\theta_0 - \delta z_0}{\alpha}\right).$$

Finally, defining $\xi_0 = (\theta_0 - \delta z_0)/\alpha$, we derive

$$E(\theta \mid \bar{\theta} \geq \theta_0, z = z_0) - \theta_0 = \alpha[E(\xi \mid \xi \geq \xi_0) - \xi_0] = \alpha\sigma_1 J(\xi_0/\sigma_1) > 0,$$

where $J(x) \equiv H(x) - x$. The term $J(\xi_0/\sigma_1)$ is equal to zero only in the limit, when $z_0 = -\infty$, given that $\delta > 0$. It is now routine work to show that if $z_0 > -\infty$, then W can be increased by reducing z_0 and increasing θ_0 slightly.

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